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PROPAGATION OF ACOUSTIC WAVES IN UNSATURATED POROUS MEDIA

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A mathematical model of propagation of acoustic waves in a porous medium saturated with a two-phase mixture is proposed. The mechanism of initiation of slow motion as a result of prolonged nondestructive acoustic action is analyzed.

The problem of the description of the nonlinear wave dynamics of rocks in the acoustic frequency range assumes not only analysis of the propagation and attenuation mechanisms for fast elastic waves [1, 2] but also estimation of slow motion (for example, filtration flow and a change in the stressed-strained state) that is initiated as a result of prolonged action of acoustic waves [3]. A mathematical model of a fluid-saturated deformable porous medium can be efficiently constructed by the methods of porous-medium mechanics [4] and, under certain conditions of the existence of dimensionless perturbations, be investigated using asymptotic methods, for example, an operator method of multiscale expansions [5].

Implementation of the indicated approach in cases of single-phase saturation is presented in [6-8], in which account for dispersion factors (viscous stresses and thermal diffusivity) is shown to give rise, for the highest spatial derivatives in the equations of the model, to dimensionless parameters: the inverse acoustic Reynolds Re_a^{-1} and Péclet Pe_a^{-1} numbers. For large $\mathrm{Re}_a >> 1$ and $\mathrm{Pe}_a >> 1$, which correspond to a high-permeability medium saturated with a low-viscosity fluid, it becomes possible to use the multiscale-expansion method for asymptotic investigation of the model in the ultrasonic range of 10^4-10^5 Hz (when the carrier-wave frequency, on the one hand, exceeds the characteristic frequency of interphase transfer of momentum $\omega_* = \nu m^2/\rho\kappa$, where ν, ρ, m, and κ are the characteristic viscosity of the fluid and density, porosity, and permeability of the medium, and, on the other, does not fall within the megahertz range, in which scattering mechanisms should be allowed for [9]).

The asymptotic method enables us to separate fast and slow motions and solve two basic problems. The first problem consists in assessing fast processes of wave propagation and attenuation against the background of the slow motion of the medium. The solution is constructed in two stages: first, linear dispersion analysis is performed, which enables us to assess the stability and regimes of wave attenuation over a wide range of acoustic Reynolds and Péclet numbers. Next, for large Re_a and Pe_a, the Cauchy problem for the initial system of equations of the laws of conservation of mass, momentum, and energy is transformed to a Cauchy problem for nonlinear evolution equations (of the Burgers type), which permits investigation of nonlinear mechanisms of attenuation.

The second problem involves evaluation of the "work" of acoustic waves over a long time, when averaging of nonlinear oscillations leads to accumulation of changes in the parameters of the background motion. Clearly, this problem is key for modeling the acoustic action of waves of different types on oil pools. Construction of the solution here assumes derivation and numerical investigation of a system of equations that describes background motion in "real" time and takes into account (in the form of sources) the nonlinear contribution of fast oscillations.

Formulation of the Pool Model. We consider a porous medium that consists of an effective viscoelastic solid phase that is formed by a thermoelastodeformable skeleton and a viscous liquid bound by the skeleton surface and is saturated with a two-phase fluid, i.e., a viscous liquid and gas that occupy their pore systems. The stress tensors in the liquid and gas phases will have the form (i, j = 1, 2, 3)

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$$P_{ij}^{\rm liq} = -p_{\rm liq}\,\delta_{ij} + \nu_{\rm liq}\,[\partial v_{{\rm liq}i}/\partial x_j + \partial v_{{\rm liq}j}/\partial x_i - (2/3)\,(\partial v_{{\rm liq}k}/\partial x_k)\,\delta_{ij}]\,,$$
$$P_{ij}^{\rm g} = -p_{\rm g}\,\delta_{ij} + \nu_{\rm g}\,[\partial v_{{\rm g}i}/\partial x_j + \partial v_{{\rm g}j}/\partial x_i - (2/3)\,(\partial v_{{\rm g}k}/\partial x_k)\,\delta_{ij}]\,. \tag{1}$$

We write the equations of state, the thermodynamic relations for the liquid and gas phases, and the capillary-equilibrium condition:

$$\rho_{\rm g} = p_{\rm g}/(RT_{\rm g})\,,\ \ E_{\rm g} = C_{\rm g}T_{\rm g}\,,\ \ \rho_{\rm liq} = \rho_{\rm liq0}\,(1 + \beta_{\rm liq}\,(p_{\rm liq} - p_{\rm liq0}) - \varphi_{\rm liq}\,(T_{\rm liq} - T_{\rm liq0})\,,$$
$$\rho_{\rm liq}dE_{\rm liq} = \rho_{\rm liq}C_{\rm liq}dT_{\rm liq} + (p_{\rm liq}/\rho_{\rm liq})\,d\rho_{\rm liq} - \varphi_{\rm liq}T_{\rm liq}dp_{\rm liq}\,,\ \ p_{\rm g} - p_{\rm liq} = \gamma J\,(s)\,. \tag{2}$$

The porous-medium skeleton and the bound liquid will be assumed to form an effective viscoelastic solid phase that exhibits the elastic properties of the skeleton and the viscous properties of the liquid. Here, the skeleton and the bound liquid have the same velocity, temperature, and pressure. With allowance for the viscoelastic properties, the rheological relation for the solid phase has the form $(i, j = 1, 2, 3)$

$$\sigma_{ij} = Ke_{kk}\delta_{ij} + 2G\,(e_{ij} - e_{kk}\delta_{ij}/3) + \beta_{\rm s}Kp\delta_{ij} - \varphi_{\rm s}KT_{\rm s}\delta_{ij} +$$
$$+ {\rm Re}_{\rm a}^{-1}\alpha m\nu_{\alpha}\,[\partial v_{{\rm s}i}/\partial x_j + \partial v_{{\rm s}j}/\partial x_i - (2/3)\,(\partial v_{{\rm s}k}/\partial x_k)\,\delta_{ij}]\,, \tag{3}$$

where the average pore pressure $p = sp_{\rm liq} + ((1 - s)p_{\rm g}$.

The equations of state and the thermodynamic relations for the components of the solid phase will be represented as

$$\rho_{\alpha} = \rho_{\alpha 0}\,(1 - \beta_{\alpha}\,(\sigma_{kk}^{\rm s}/3 - \sigma_0) - \varphi_{\alpha}\,(T_{\rm s} - T_{\rm s0}))\,;$$
$$\rho_{\rm s} = \rho_{\rm s0}\,(1 - \beta_{\rm s}\,(\sigma_{kk}^{\rm s}/3 - \sigma_0) - \varphi_{\rm s}\,(T_{\rm s} - T_{\rm s0}))\,;$$
$$\rho_{\rm s}dE_{\rm s} = \rho_{\rm s}C_{\rm s}dT_{\rm s} + \sigma_{ij}^{\rm s}de_{ij} + \varphi_{\rm s}T_{\rm s}d\sigma_{kk}^{\rm s}/3\,; \tag{4}$$
$$\rho_{\alpha}dE_{\alpha} = \rho_{\alpha}C_{\alpha}dT_{\rm s} - \sigma_{kk}^{\rm s}/(3\rho_{\alpha})\,d\rho_{\alpha} + \varphi_{\alpha}T_{\rm s}d\,\sigma_{kk}^{\rm s}/3\,,$$

where the true stresses in the solid phase are determined by the relation

$$\sigma_{ij}^{\rm s} = \sigma_{ij}/(1 - (1 - \alpha)\,m) + sP_{ij}^{\rm liq} + (1 - s)\,P_{ij}^{\rm g}\,.$$

We introduce the following dimensionless variables and parameters:

$$x' = x/x_0\,,\ \ t' = t/t_0\,,\ \ u' = u/x_0\,,\ \ \rho' = \rho/\rho_0\,,\ \ P' = P/K_0\,,\ \ \sigma' = \sigma/K_0\,,\ \ T' = T/\theta_0\,,$$
$$v' = v/v_0\,,\ \ \beta' = \beta K_0\,,\ \ \varphi' = \varphi\theta_0\,,\ \ K' = K/K_0\,,\ \ G' = G/K_0\,,\ \ E' = E/v_0^2\,,$$
$$\nu' = \nu/(K_0t_0)\,,\ \ C' = C\theta_0/v_0^2\,,\ \ \lambda' = \lambda\theta_0t_0/(x_0^2v_0^2\rho_0)\,,\ \ \chi' = \chi\theta_0t_0/(v_0^2\rho_0)\,,$$
$$R' = R\theta_0/v_0^2\,,\ \ k' = kx_0\,,\ \ \omega' = \omega t_0\,,$$

where $x_0 = v_0t_0$, $t_0 = \rho_0\kappa/\nu_{\rm g}$, $v_0 = (K_0/\rho_0)^{1/2}$.

Dropping the primes on the symbols, we write, in dimensionless form, the system of mass, momentum, and energy equations $(i, j = 1, 2, 3)$.

The laws of conservation of mass are

$$\partial (m (1-s) \rho_g)/\partial t + \nabla_x (m (1-s) \rho_g v_g) = 0; \quad \partial (ms\rho_{liq})/\partial t + \nabla_x (ms\rho_{liq} v_{liq}) = 0;$$
$$\partial (am\rho_\alpha + (1-m)\rho_s)/\partial t + \nabla_x ((am\rho_\alpha + (1-m)\rho_s) v_s) = 0, \tag{5}$$

where $v_{liq} = (v_{liq1}, v_{liq2}, v_{liq3})$, $v_g = (v_{g1}, v_{g2}, v_{g3})$, $v_s = (v_{s1}, v_{s2}, v_{s3})$.

The laws of conservation of momentum are

$$\rho_{liq} [\partial/\partial t + \langle v_{liq}, \nabla_x\rangle] v_{liqi} - \partial P_{ij}^{liq}/\partial x_j + ms (1-\alpha) (v_{liqi} - v_{si})/f_{liq}(s) = 0;$$

$$\rho_g [\partial/\partial t + \langle v_g, \nabla_x\rangle] v_{gi} - \partial P_{ij}^{g}/\partial x_j + m (1-s)(1-\alpha)(v_{gi} - v_{si}) v_g/(v_{liq} f_g(s)) = 0;$$

$$(am\rho_\alpha + (1-m)\rho_s) [\partial/\partial t + \langle v_s, \nabla_x\rangle] v_{si} - \partial\sigma_{ij}/\partial x_j -$$
$$- s(1-(1-\alpha)m)\partial P_{ij}^{liq}/\partial x_j - (1-s)(1-(1-\alpha)m)\partial P_{ij}^{g}/\partial x_j -$$
$$- m^2 s^2 (1-\alpha)^2 (v_{liqi} - v_{si})/f_{liq}(s) - m^2 (1-s)^2 (1-\alpha)^2 (v_{gi} - v_{si}) v_g/v_{liq} f_g(s)) = 0. \tag{6}$$

The laws of conservation of energy are

$$m(1-s)(1-\alpha)\rho_g [\partial/\partial t + \langle v_g, \nabla_x\rangle] E_g - ms(1-\alpha) P_{ij}^{g} \partial v_{gi}/\partial x_j + \chi_g (T_g - T_s) -$$
$$- m^2 s^2 (1-\alpha)^2 (v_g - v_s)^2 v_g/(v_{liq} f_g(s)) - (1-\alpha)\lambda_g \nabla_x (m(1-s)\nabla_x T_g) = 0;$$

$$ms(1-\alpha)\rho_{liq} [\partial/\partial t + \langle v_{liq}, \nabla_x\rangle] E_{liq} - ms(1-\alpha) P_{ij}^{liq} \partial v_{liqi}/\partial x_j -$$
$$- m^2 s^2 (1-\alpha)^2 (v_{liq} - v_s)^2/f_{liq}(s) + \chi_{liq}(T_{liq} - T_s) - (1-\alpha)\lambda_{liq}\nabla_x (ms\nabla_x T_{liq}) = 0;$$

$$(1-m)\rho_s [\partial/\partial t + \langle v_s, \nabla_x\rangle] E_s + am\rho_\alpha [\partial/\partial t + \langle v_s, \nabla_x\rangle] E_\alpha -$$
$$- [\sigma_{ij} + s(1-(1-\alpha)m) P_{ij}^{liq} + (1-s)(1-(1-\alpha)m) P_{ij}^{g}] \partial v_{si}/\partial x_j -$$
$$- \chi_{liq}(T_{liq} - T_s) - \chi_g (T_g - T_s) - \nabla_x ((1-m)\lambda_s + am\lambda_\alpha)\nabla_x T_s = 0 \tag{7}$$

and

$$\partial u_i/\partial t - v_{si} = 0; \quad e_{ij} - [\partial u_i/\partial x_j + \partial u_j/\partial x_i]/2 = 0. \tag{8}$$

Substitution of the rheological and thermodynamic relations into the system of conservation laws (1)-(8) gives rise, for the second spatial derivatives in the momentum and energy equations, to the inverse acoustic Reynolds Re_a^{-1} and Péclet Pe_a^{-1} numbers, respectively. It is easy to see that

$$\mathrm{Re}_a = v_0 x_0 \rho_0/\nu_0 \equiv K_0 \rho_0 \kappa/\nu_0^2, \quad \mathrm{Pe}_a = v_0 x_0 \rho_0 C_0/\lambda_0 \equiv K_0 \rho_0 C_0 \kappa/(\nu_0 \lambda_0).$$

Estimates of the dimensionless parameters can be obtained using the characteristic constants of rocks and pore fluids: $K_0 \sim 10^8 - 10^9$ Pa, $\theta_0 \sim 10^2 - 10^3$ K, $\rho_0 \sim 10^1 - 10^3$ kg/m^3, $\beta \sim 10^{-10} - 10^{-9}$ Pa^{-1}, $\varphi \sim 10^{-6} - 10^{-3}$ K^{-1}, $C_0 \sim 10^3$ J/(kg·K), $\lambda_0 \sim 10^{-3} - 10^0$ W/(m·K), $\nu_0 \sim 10^{-5} - 10^{-3}$ Pa·sec, and $\kappa \sim 10^{-15} - 10^{-12}$ m^2. For these parameters, we have an Re_a range of $\sim 10^0 - 10^6$ and a Pe_a range of $\sim 10^1 - 10^6$. Thus, in the actual range of pool parameters, the degree of influence of dispersion factors (viscous stresses and thermal diffusivity) can prove to be both rather small (in a high-permeability medium saturated with a low-viscosity fluid) and finite (in a low-permeability medium saturated with a high-viscosity fluid). For large $\mathrm{Re_a} \gg 1$ and $\mathrm{Pe_a} \gg 1$, it becomes possible to identify perturbations associated with these numbers and to investigate the process asymptotically.

Evolution of Small Free Oscillations. To investigate the special features of wave propagation in a porous medium saturated with a two-phase liquid-gas mixture, in what follows we restrict ourselves to the Cauchy problem with the initial data $(i, j = 1, 2, 3)$

$$u_i|_{t=0} = \overset{0}{u_i}, \quad v_{si}|_{t=0} = \overset{0}{v_{si}}, \quad v_{gi}|_{t=0} = \overset{0}{v_{gi}}, \quad v_{\mathrm{liq}i}|_{t=0} = \overset{0}{v_{\mathrm{liq}i}}, \quad m|_{t=0} = \overset{0}{m}, \quad s|_{t=0} = \overset{0}{s},$$

$$p_{\mathrm{liq}}|_{t=0} = \overset{0}{p_{\mathrm{liq}}}, \quad p_{\mathrm{g}}|_{t=0} = \overset{0}{p_{\mathrm{g}}}, \quad T_{\mathrm{g}}|_{t=0} = \overset{0}{T_{\mathrm{g}}}, \quad T_{\mathrm{liq}}|_{t=0} = \overset{0}{T_{\mathrm{liq}}}, \quad T_{\mathrm{s}}|_{t=0} = \overset{0}{T_{\mathrm{s}}};$$

$$e_{ij}|_{t=0} \equiv \overset{0}{e_{ij}} = [\partial \overset{0}{u_i}/\partial x_j + \partial \overset{0}{u_j}/\partial x_i]/2;$$

$$\sigma_{ij}|_{t=0} \equiv \overset{0}{\sigma_{ij}} = K\overset{0}{e_{kk}}\delta_{ij} + 2G(\overset{0}{e_{ij}} - (1/3)\overset{0}{e_{kk}}\delta_{ij}) + \beta_s K \overset{0}{p}\delta_{ij} -$$

$$- \varphi_s K \overset{0}{T_{\mathrm{s}}}\delta_{ij} + \alpha \overset{0}{m}\nu_\alpha [\partial \overset{0}{v_{si}}/\partial x_j + \partial \overset{0}{v_{sj}}/\partial x_i - (2/3)(\partial \overset{0}{v_{sk}}/\partial x_k)\delta_{ij}]. \tag{9}$$

The solution to problem (1)-(9) will be sought in the form of a superposition of the slow background motion and its small rapidly-oscillating perturbation

$$U = U_{\mathrm{b}}(x, t) + 0 U_1 \exp(iS/0). \tag{10}$$

Here ε is the perturbation, $U = (m, v_{gi}, v_{\mathrm{liq}i}, v_{si}, p_{\mathrm{liq}}, p_{\mathrm{g}}, s, T_{\mathrm{g}}, T_{\mathrm{liq}}, T_{\mathrm{s}}, u_i, \sigma_{ij}, e_{ij})$ is the vector-function of the sought quantities, and the phase is $S/\varepsilon = \langle k\xi \rangle - \omega t$, where the fast variables are $\tau = t/\varepsilon$ and $\xi_i = x_i/\varepsilon$.

For simplicity we take, as the background solution, a uniform steady state, i.e., a quiescent state $U_{\mathrm{b}}^{(0)}$ whose existence is governed by the conditions of zero velocities of phase motion ($v_{si}^{(0)} = v_{gi}^{(0)} = v_{\mathrm{liq}i}^{(0)} = 0$), equality of phase temperatures ($T_{\mathrm{g}}^{(0)} = T_{\mathrm{s}}^{(0)} = T_{\mathrm{liq}}^{(0)} = T^{(0)}$), and independence of the constants $m^{(0)}$, $p_{\mathrm{g}}^{(0)}$, $p_{\mathrm{liq}}^{(0)}$, $s^{(0)}$, $T^{(0)}$, $u_i^{(0)}$ of x.

The system of equations (1)-(8) linearized against the background $U_{\mathrm{b}}^{(0)}$ can be represented in the operator form

$$A \cdot U = 0, \tag{11}$$

where

$$A(\partial/\partial t, \partial/\partial x_i, \partial^2/\partial x_i \partial x_j, U_0) = A_1(\partial/\partial t, \partial/\partial x_i, U_{\mathrm{b}}^{(0)}) + A_2(\partial^2/\partial x_i \partial x_j, U_{\mathrm{b}}^{(0)}) + A_3(U_{\mathrm{b}}^{(0)}).$$

In what follows, let U_1 be determined by the amplitude of the initial perturbation, i.e., the oscillating portion $U^{(0)} = (m^{(0)}, v_{gi}^{(0)}, v_{\mathrm{liq}i}^{(0)}, v_{si}^{(0)}, p_{\mathrm{liq}}^{(0)}, p_{\mathrm{g}}^{(0)}, s^{(0)}, T_{\mathrm{g}}^{(0)}, T_{\mathrm{liq}}^{(0)}, T_{\mathrm{s}}^{(0)}, u_i^{(0)}, \sigma_{ij}^{(0)}, e_{ij}^{(0)})$, the wave-vector components are real numbers, and the frequencies ω can be complex. This approximation corresponds to the Cauchy problem for the evolution of k-waves that are the analog of free oscillations [10]. Here, the imaginary part of ω determines the coefficient of wave attenuation.

Substitution of the solution in the form of (10) into the linear system (11) leads to the condition of nontrivial solvability: a dispersion equation that relates ω to k. For analyzing the attenuation mechanisms for waves of different types, it is of interest to investigate the dependences of the phase velocities $V(k) = \mathrm{Re}(\omega/|k|)$ and the attenuation coefficients $\delta(k) = \mathrm{Im}(\omega)$ in various scales of the acoustic Reynolds and Péclet numbers and the related perturbation ε. Results under the assumption of the equality of the Froude number $\mathrm{Fr} = 1$ are given below for three cases. We note that the wave characteristics obtained as functions of $|k|$ correspond to an isotropic background stressed-strained state.

Analysis of Dispersion Relations. *Case 1*: $\varepsilon^2 = \mathrm{Re_a^{-1}} = \mathrm{Pe_a^{-1}} \sim 10^{-6}$. It is easy to see that substitution of (10) into (11) leads, accurate to $O(\varepsilon)$, to the dispersion relation

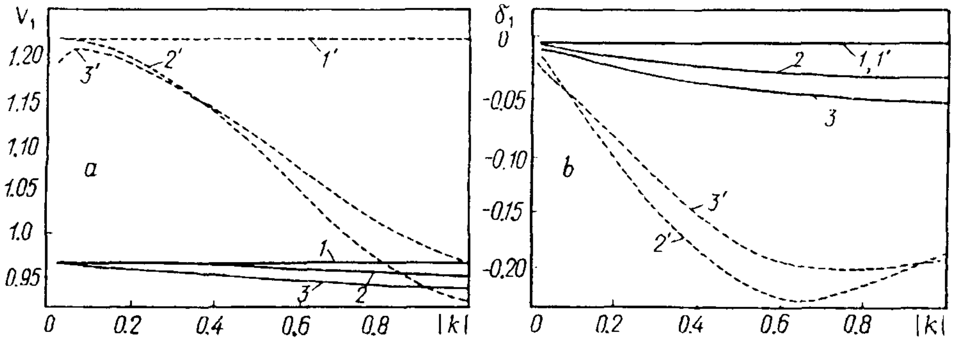

Fig. 1. Dimensionless phase velocity V_1 (a) and attenuation coefficient δ_1 (b) of a longitudinal wave of the 1st kind vs. the modulus of the wave vector.

$$J_1 = \text{Det}\, A_1\, (-i\omega\,,\, ik_i\,,\, U_b^{(0)}) = 0\,. \tag{12}$$

The dispersion relations are not given in explicit form because of their extremely cumbersome nature. Further investigation is limited to a numerical analysis with graphical representation of the results.

In the case under study, the system of equations of the model is hyperbolic, and dissipation and dispersion effects are not allowed for in the linear analysis. This leads to absence of attenuation ($\text{Im}(\omega) = 0$) and constant velocities for all types of waves for any k. We note that linear analysis is the first stage of using the multiscale-expansion method to construct an asymptotic solution of higher accuracy. Thus, constructing a solution accurate to $O(\varepsilon^2)$ leads to a nonlinear equation of the Burgers type, and the next step of the expansion (accurate to $O(\varepsilon^3)$ leads to a Korteweg–de Vries–Burgers-type equation. This nonlinear evolution equation already allows for effects of weak dissipation and dispersion and enables us to analyze nonlinear regimes of attenuation of waves and their resonance interaction [11].

Case 2: $\varepsilon = \text{Re}_a^{-1} = \text{Pe}_a^{-1} \sim 10^{-3}$. Substituting (10) into (11) leads, accurate to $O(\varepsilon)$, to the dispersion relation

$$J_2 = \text{Det}\,[A_1\,(-i\omega\,,\, ik_i\,,\, U_b^{(0)}) + A_2\,(i^2 k_i k_j\,,\, U_b^{(0)})] = 0\,. \tag{13}$$

As can be seen from (13), in the case under study the dispersion factors are allowed for and dissipation due to interphase friction is disregarded in the characteristic equation. The system of equations of the model is no longer hyperbolic, which is expressed, in particular, in the complex-valued nature of the frequencies of all the identified wave types. Here it is of interest to evaluate in a linear approximation the phase velocities of the waves V and the attenuation coefficients δ as functions of the wavelength.

Asymptotic analysis of this case in a nonlinear statement assumes the use of a modified multiscale-expansion method [5] for the case of "strong" dispersion, which is beyond the scope of the present work.

Case 3: $\varepsilon = \text{Re}_a^{-1} = \text{Pe}_a^{-1} \sim 1$. Here all the factors (inertia, dissipation, and dispersion factors) prove to be of the same order, and the system of equations of the model does not involve a perturbation that governs the scale of fast variables ($\tau \equiv t$, $\xi_i \equiv x_i$). To analyze the evolution of perturbations of the background solution, let us assume additionally that $U_1 << U_b$. Substitution of (10) into (11) leads to the dispersion relation

$$J_3 = \text{Det}\,[A_1\,(-i\omega\,,\, ik_i\,,\, U_b^{(0)}) + A_2\,(i^2 k_i k_j\,,\, U_b^{(0)}) + A_3\,(U_b^{(0)})] = 0\,. \tag{14}$$

In the case under study, it becomes possible to assess the joint effect of dissipation and dispersion mechanisms.

Figures 1-3 present results of calculations of the phase velocities V_i and the attenuation coefficients δ_i for longitudinal waves of the 1st ($i = 1$) and 2nd ($i = 2$) kind and capillary waves ($i = 3$) based on a numerical determination of the roots of dispersion relations (12)-(14). The numbers of the presented curves correspond to the above *cases*. The calculations are performed for the following dimensionless parameters: $\nu_g = 0.01$, $\nu_{liq} = 0.1$, $\nu_\alpha = 1$, $\rho_{\alpha 0} = 1$, $\rho_{liq0} = 1$, $\rho_{s0} = 2.5$, $\varphi_\alpha = 0.5$, $\beta_\alpha = 0.5$, $\varphi_{liq} = 0.5$, $\beta_{liq} = 0.5$, $\varphi_s = 0.1$, $\beta_s = 0.1$, $G = 0.3$, $K = 1$,

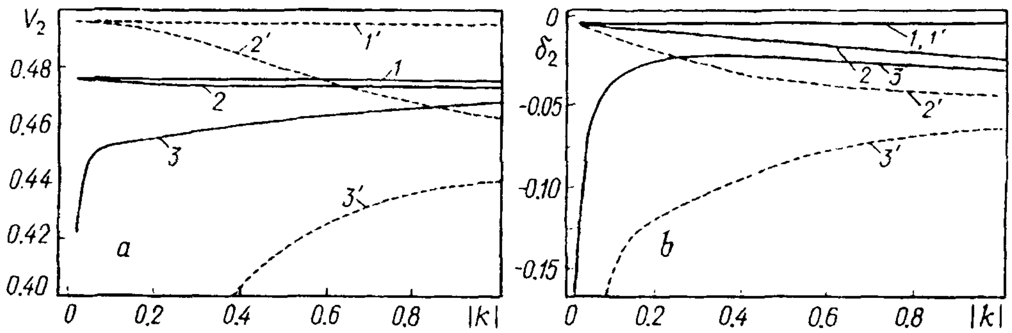

Fig. 2. Dimensionless phase velocity V_2 (a) and attenuation coefficient δ_2 (b) of a longitudinal wave of the 2nd kind vs. the modulus of the wave vector.

$C_\alpha = 1$, $C_{liq} = 1$, $C_g = 0.3$, $C_s = 0.4$, $\lambda_g = 0.001$, $\lambda_{liq} = 0.5$, $\lambda_\alpha = 0.5$; $\lambda_s = 1$, $\chi_{liq} = 0.1$, $\chi_g = 0.01$, $\alpha = 0.1$, $m^{(0)} = 0.2$, $T^{(0)} = 1$, $p^{(0)}_{liq} = 0.1$, $e^{(0)}_{ij} = 0$, $J = 0.5(1 + (1 - 2s)^9)$, $f_{liq} = s^2$, $f_g = (1 - s)^2$, $\gamma = 0.001$.

The solid curves (1, 2, and 3) are obtained for $s^{(0)} = 0.2$, and the dashed curves (1′, 2′, and 3′) correspond to $s^{(0)} = 0.8$.

Let us first consider longitudinal waves of the 1st kind (Fig. 1). *Case 1* (curves 1 and 1′) is characterized by a wavelength-independent phase velocity and absence of attenuation (this is true of all the wave types for *case 1*). Comparison of the solid and dashed curves demonstrates a significant increase in the propagation velocity for waves of the 1st kind with increase in the liquid-phase content. A tendency for a decrease in the velocity of the wave with decrease in its wavelength when the effect of the dispersion and dissipation factors shows up is also evident (curves 2 (2′) and 3 (3′) in Fig. 1a). However, velocity dispersion becomes substantial only as the saturation $s^{(0)}$ increases.

Attenuation of waves of the 1st kind increases as a whole as the wavelength decreases (curves 2 (2′) and 3 (3′) in Fig. 1b). With high saturation of the liquid phase, there is an attenuation extremum, where the tendency for an increase in the absolute value of the attenuation coefficient gives way to the opposite (curves 2 and 2′ in Fig. 1b).

An ambiguous effect of the magnitude of the saturation $s^{(0)}$ shows up clearly in the example of longitudinal waves of the 2nd kind that are distinguished by oppositely directed motions of the solid and fluid phases. As is evident from a comparison of curves 2 and 2′ in Fig. 2 in *case 2*, for a larger $s^{(0)}$ the phase velocity proves to be higher only in the longwave range, while the attenuation increases with $s^{(0)}$ for all wavelengths. *Case 3*, where interphase friction has a determining effect, is characterized by a sharp decrease in the wave velocity and an increase in the attenuation as the wavelength decreases. This is in good agreement with known experimental facts of strong attenuation of waves of the 2nd kind in rocks [3].

It is significant that in the considered case of saturation of the porous medium with a two-phase fluid (a liquid-gas mixture), the absolute values of the attenuation coefficients for waves of the 1st and 2nd kind prove to be of the same order of magnitude in the shortwave (ultrasonic) range. An important difference shows up in the magnitude of the phase velocity.

We analyze capillary waves that are characterized by antiphase motion of the solid phase and the liquid–gas mixture in them.

The curves of Fig. 3 indicate very low velocities of the longitudinal capillary waves that differ from the velocities of other longitudinal waves by 1-2 orders of magnitude. With low saturation $s^{(0)}$, the effect of the dispersion and dissipation factors leads to near-zero phase velocities (curves 2 and 2′ in Fig. 3a). As $s^{(0)}$ increases, an extremum character is observed for the dependence of V_3 in the longwave range.

The character of the curves of Fig. 3b indicates a qualitative similarity of the dependences of the attenuation coefficients for capillary waves and waves of the 2nd kind. The important difference is in the fact that, in the capillary waves, the attenuation coefficient is an order of magnitude higher.

Assessment of the Resultant Action of Acoustic Waves. For asymptotic analysis of the dynamics of the background motion, we represent the solution to problem (1)-(9) in the form [6-8, 11]

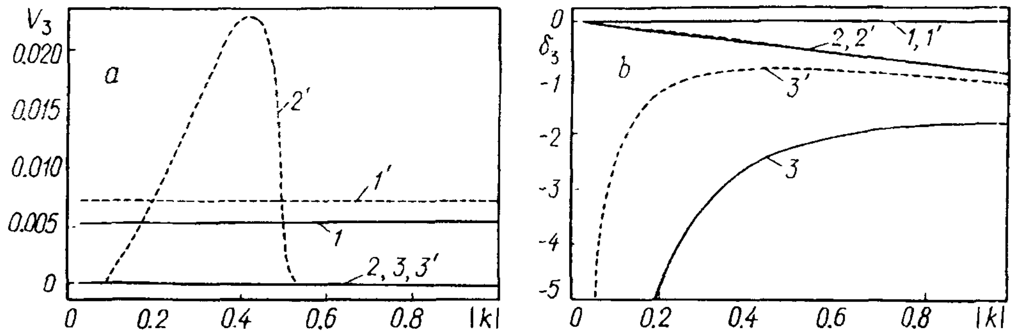

Fig. 3. Dimensionless phase velocity V_3 (a) and attenuation coefficient δ_3 (b) of a longitudinal capillary wave vs. the modulus of the wave vector.

$$U = U_b(x, t) + U^{(1)}(\xi, \tau, x, t), \quad U^{(1)} = \varepsilon H W^{(1)} + \varepsilon^2 H W^{(2)} + \dots, \quad U_b = U_b^{(0)} + \varepsilon H W_b^{(1)} + \varepsilon^2 H W_b^{(2)} + \dots, \tag{15}$$

where $H(x, t)$ is the null vector of the matrix of the operator A_1, linearized against the background $U_b^{(0)}$, that corresponds to one of the frequencies $\omega_i(x, t, k)$ identified as a result of dispersion analysis.

The solution for U_b is constructed by substituting (15) into (1)-(8) and collecting one-scale terms. The final system of equations admits, as a zero approximation of the background solution, any constant vector function $U_b^{(0)}$ = const. For the first approximation $W_b^{(1)}$ of the background solution (accurate to $O(\varepsilon)$, we obtain a homogeneous hyperbolic equation, but since the initial data (9) mod $O(\varepsilon)$ are zero and the function $W^{(1)}$ is a function with a zero average, this equation has a unique zero solution $W_b^{(1)} = 0$.

Next, for $W_b^{(2)}(x, t)$ (a solution accurate to $O(\varepsilon^2)$, we obtain the following equation:

$$\frac{dW_b^{(2)}}{dt_A} + K_1 W_b^{(2)} + K_2 = 0, \tag{16}$$

where d/dt_A is the total derivative along the wave trajectory; the coefficients K_1 and K_2 are determined upon substitution of the form of the solution into the initial system of equations; K_2 involves "sources" – integral terms that are averages of nonlinear terms (squares of fast wave functions $W^{(1)}$).

For the linear problem, the condition of zero averages would lead to a homogeneous equation that has only a trivial solution. In the nonlinear case under study, even with a zero initial condition for the function $W_b^{(2)}(x, t)$, which is a consequence of the initial data (9), the inhomogeneous equation (16) will have a nontrivial solution, which explains the mechanism of initiation of the medium's slow motion as being a result of nonlinear interphase interaction (friction).

We next construct an approximate solution of the plane traveling wave type. Substitution of $S = kx - \omega t$ leads to $d/dt_A = 0$, and the solution has the form

$$W_b^{(2)} = -\frac{K_2}{K_1} \tag{17}$$

Figure 4 presents results of calculations for longitudinal waves of the 1st and 2nd kind and capillary waves. The function $W_b^{(2)}$ is plotted against the initial saturation of the liquid phase in the pool $s^{(0)}$ for the above dimensionless parameters, $k = (1, 0, 0)$, and a constant amplitude $W^{(1)} = 1$.

As Fig. 4 shows, the action of waves of the 1st kind and capillary waves depends substantially on the background saturation. This shows up especially strongly for capillary waves, which can have a substantial effect at low saturation of one phase, i.e., where the gradient of the capillary forces is maximum. Conversely, the "work" of waves of the 2nd kind practically does not depend on the pool saturation.

Naturally it is of interest to elucidate motion of what scale can be induced by small-amplitude acoustic waves. As estimates obtained by reducing results of calculations to a dimensional form show, prolonged ultrasonic action with an amplitude of ~0.1 MPa initiates filtration motion with velocities of ~ 10^{-6}–10^{-5} m/sec, which are

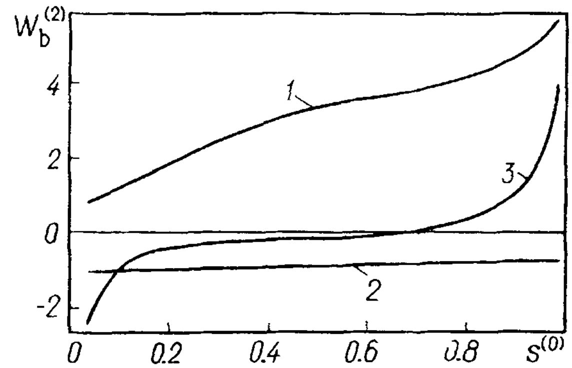

Fig. 4. Change in the background state $W_b^{(2)}$ vs. the initial saturation of the liquid phase in a stratum $s^{(0)}$ under the action of longitudinal waves of the 1st (curve 1) and 2nd (curve 2) kind and capillary waves (curve 3).

quite comparable with the velocities of the inflow to the well in working oil- and gas-bearing strata. We note that there is a quadratic dependence of the amplitude of the resultant motion on the amplitude of the acoustic action.

NOTATION

x, spatial coordinate; t, time; m, porosity; ρ, density; ν, viscosity; p, pressure; T, temperature; α, amount of bound liquid; u, displacement vector; e_{ij}, deformation tensor; σ_{ij}, effective-stress tensor; f, phase permeability; E, specific internal energy; K, modulus of elasticity; G, shear modulus; R, gas constant; C, heat capacity; β, compressibility factor; φ, coefficient of thermal expansion; J, Leverette function; γ, dimensionless coefficient of surface tension. Subscripts: liq, liquid phase; g, gas phase; s, solid phase; α, bound liquid.

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